# **A Mesh Deformation Algorithm and Its Application in Optimal Motor Design**

Lin Yang, S. L. Ho, W. N. Fu, and Lei Liu

Department of Electrical Engineering, The Hong Kong Polytechnic University, Hong Kong

eewnfu@polyu.edu.hk

**In this paper, a remesh-free deformation method is reported to speed up the modeling of electric motor (EM) in optimal design. Neither the mesh regeneration nor an increase in the number of unknowns is required for the proposed method. Refined meshes can be quickly derived using a coordinate mapping technique. The calculation time can therefore be greatly reduced when the geometric parameters are changed during the optimal design process. At the same time, the mesh quality is guaranteed to ensure the FEM calculation is accurate. Numerical results are reported to demonstrate the efficiency and effectiveness of the proposed method.**

*Index Terms***—Electric motor, finite element method, mesh deformation, optimization.**

## I. INTRODUCTION

PTIMAL DESIGN of electric motor (EM) is generally **O** PTIMAL DESIGN of electric motor (EM) is generally complex and time-consuming, since it is intervening with geometrical changes in the parameters of the device during the optimization process [1-3]. For numerical simulations based on finite element method (FEM), a conforming mesh with good quality as well as suitable mesh density is needed in the pre-processing phase. Usually, constructing an acceptable mesh is complicated and time-consuming, especially in complex problems such as in the parametric optimization of motors. The computing time for mesh generation is generally excessively long and sometimes it might take up more than 80% of the total computing time [4]. Robust and simple mesh deformation methods that can reduce the total optimization time are therefore particularly meaningful.

In this paper, a robust and simple mesh deformation technique is proposed for fast and accurate EM optimization studies. Compared with existing mesh deformation methods, the solution of the partial differential equations (e.g. Laplace equation) [1] or radial basis functions [5] is avoided. Consequently the computation complexity as well as its computational cost is relatively low. In addition, the proposed method is applicable to any existing mesh without considering the mesh generation method. Great flexibility is thus being factored into the implementation of the proposed method.

### II.PROPOSED MESH DEFORMATION TECHNOLOGY

A novel mesh morphing method is proposed for fast and robust mesh deformation, both in two-dimensional (2-D) and three-dimensional (3-D) cases with simple implementation procedure.

Firstly, a constrained boundary mesh is constructed for a set of fine mesh, which covers all movable nodes in the fine mesh and serves as a deformable skeleton during mesh morphing. Selection of the boundary mesh is simple and does not need to conform exactly to the outline geometry of the device.

Secondly, the area coordinates of all the nodes in the fine mesh with respect to the boundary mesh are calculated and saved by applying a coordinate mapping technique, in which the coordinates of each node in the fine mesh can be expressed as a function (area coordinates) of the elements in the boundary mesh. This process is required once and only once. When the mesh needs to be updated due to changes in geometry, one only needs to reset the coordinates of the nodes in the boundary mesh. The new fine mesh can then be updated according to the new boundary mesh and the area coordinates which remain unchanged.

# III. COORDINATE MAPPING TECHNIQUE

Coordinate mapping is the key underlying technique for the proposed remesh-free method. It is used to determine the relative locations of the nodes of the fine mesh inside the boundary mesh. In this paper, the barycentric coordinate (also known as area coordinates in the context of a triangle) mapping approach is applied because it is robust and easy to implement. One can however use other mapping methods in a similar way.

Assume  $X_1$ ,  $X_n$  are the vertices of an element *e* (a triangle, tetrahedron, etc.) in the boundary mesh. Then for another point P in *e*, if the following equation (1) is satisfied and with at least one of the coefficients  $(a_1, \ldots, a_n)$  not being equal to zero, the coefficients are called area coordinates of P with respect to  $X_1$ ,...,  $X_n$ .

$$
P = a_1 X_1 + \dots + a_n X_n \tag{1}
$$

When the restriction  $\sum a_i = 1$  is imposed, the corresponding area coordinates are uniquely determined.

With respect to a triangle, the barycentric coordinates can be considered as proportional to the areas of triangles constructed by the vertices and the given point P, hence the barycentric coordinates are also called area coordinates.





Fig. 1. The model and parameters for: (a) The initial model; (b) The deformed model. Both the fine mesh and the boundary mesh (red solid lines) for the (c) Initial model; and (d) The deformed model.

## IV. NUMERICAL EXPERIMENTS

Firstly, the proposed remesh free method is introduced by a simple 2-D deformable object as shown in Fig. 1. The inner square is assumed to be enlarged as shown in the figure. The boundary mesh can be arbitrarily chosen as long as the boundary mesh can exactly govern all the movable nodes (as shown in Figs. 1(c) and (d) with red lines). The new mesh can be updated according to the new boundary mesh and the unchanged area coordinates (as shown in Fig. 1 (d)).

Secondly, the method is expanded to a 3-D case as shown in Fig. 2. There is a cubic object inside a ball and the inner cube is assumed to be deformed into a smaller one as shown in Figs.  $2(a)$  to  $2(d)$ .



a b

38442 elements is taken as an illustrating example. Two geometric parameters, namely the depth of PMs and the width of the teeth, are assumed to be changed during the optimization process. The enlarged view of the newly updated meshes are given in Fig. 3 and Fig. 4, respectively. One can clearly see that the mesh has been updated without changing the mesh connection relationship.

In these two illustrating examples, the average CPU time needed to compute the area coordinates of the nodes in the fine mesh is about 0.0478s and 0.0011s is needed for updating the mesh by mapping back these coordinates to the Cartesian ones. This is to be compared to about 8s which is required to regenerate such a mesh. Clearly, significant computational time is saved.



Fig. 3. (a) The enlarged view of the initial fine mesh. (b) The enlarged view of the fine mesh when the depth of PMs has been changed.





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The proposed algorithm is a robust and general mesh deformation method. More complicated cases will be introduced in the full paper to showcase the efficiency and effectiveness of the proposed method. An existing fine mesh of a permanent magnet (PM) motor with 23049 nodes and